Three-Dimensional Scattering of Internal Waves Off a Uniformly Sloping Bottom

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LONG-TERM GOAL

Any theory based on monochromatic linear wave dynamics is too simple and misleading to properly interpret or predict anything that involves a continuum of frequencies like a pulse of wave energy. Through hard case studies of scattering events of realistic non-random internal wavefields an important step will be taken towards developing an understanding of and capability to predict internal wavefield characteristics near sloping bottom topography and the generation of near-slope geostrophic currents due to the interaction of such wavefields with bottom topography.

OBJECTIVES

My objective is to determine what flows result in the vicinity of a uniformly sloping bottom when a three-dimensional realistic wave field encounters that slope. Present knowledge of such events are based on linear wave theory where single monochromatic waves are considered. In the case of constant buoyancy frequency *N* a single wave with frequency ω_i propagates at a fixed angle θ with the vertical with group velocity U_g and is either reflected forward or backward depending on whether $\omega_i < \omega_c$ or $\omega_i > \omega_c$ where ω_c is the critical frequency. At the critical frequency, where the angle θ associated with ω_i is equal to the angle of the slope, the group velocity of the outgoing wave goes to zero and wave-energy cannot propagate away. Amplitudes for the reflected wave become infinitely large and wavelengths go to zero. When a wave is reflected upslope, the propagation direction turns toward the upslope direction. These theories (Phillips 1963 and Eriksen 1982) consider steady state situations with wavefields that extent to infinity in all directions. There are no events with a 'beginning' and an 'end'. Neither are these theories sufficient to properly interpret Oceanic data. It is my objective to determine what happens when a localized wavefield, both in space and time (a wave-packet or pulse), meets a sloping bottom. I envision such a field to be generated by surface forcing of finite duration and horizontal scale.

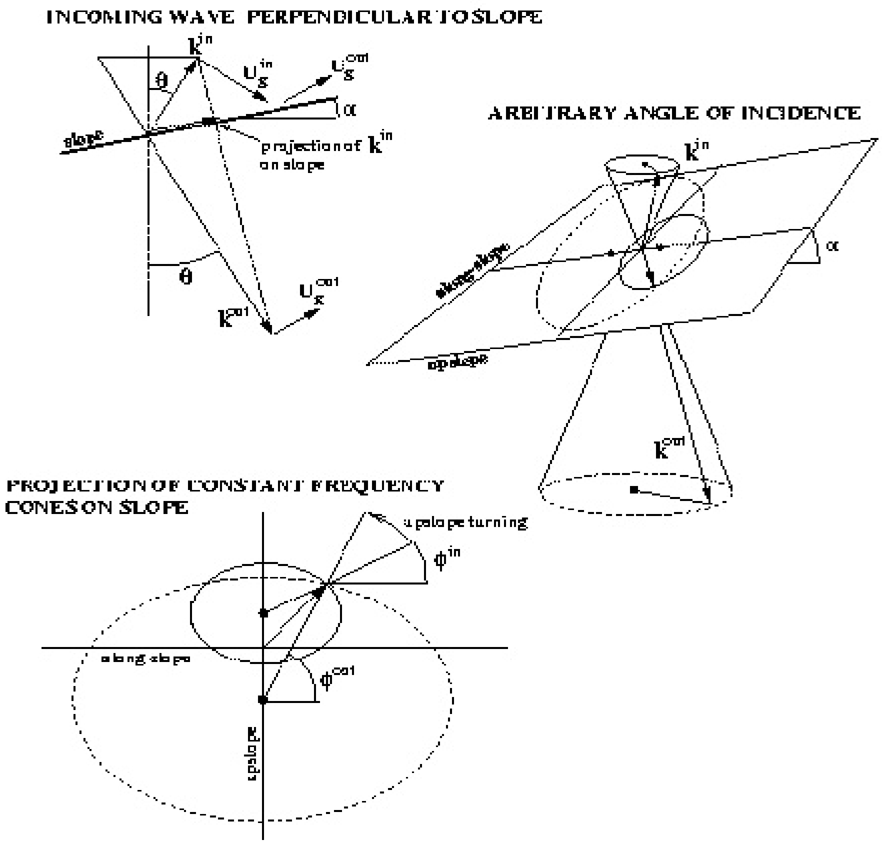


Diagram showing how to solve the reflection problem for a single monochromatic wave. This is for $\omega_i > \omega_c$. ***The along-slope component of the wavevector* k *of the incoming wave has to be matched by that of the outgoing wave. The problem reduces to a geometrical problem of finding the intersection point of two ellipses which are the projections of constant frequency cones on the slope.***

APPROACH

Surface forcing is modeled by imposing a vertical velocity field at the surface which results from the divergence of a viscous Ekman layer generated by surface winds The vertical Ekman pumping w_s at the base of the surface Ekman layer perturbs the underlying stratified Ocean. The response is investigated with the linearized dynamics, the f-plane and Boussinesq approximation and a model stratification of constant N. First the Green's function $\mathbf{G^s}$ for the semi-infinite domain is determined. This gives the response to surface pumping infinitely concentrated at one position in space and time. The response of the fluid to arbitrary pumping w_s is then given by the convolution $\mathbf{G^s}$ow_s where 'o' stands for the convolution integral over the surface and the time history of the surface pumping. The next step is to determine the Green's function $\mathbf{G^n}$ for the response to a given normal flow u_n at the slope. If we take for u_n the normal component of the velocity field $\mathbf{u}$=$\mathbf{G^s}$ow_s at the slope, then -$\mathbf{G^n}$ou_n gives the reflected wave field. The velocity field due to both the incoming and reflected wave field is then $\mathbf{u}$=$\mathbf{G^s}$o

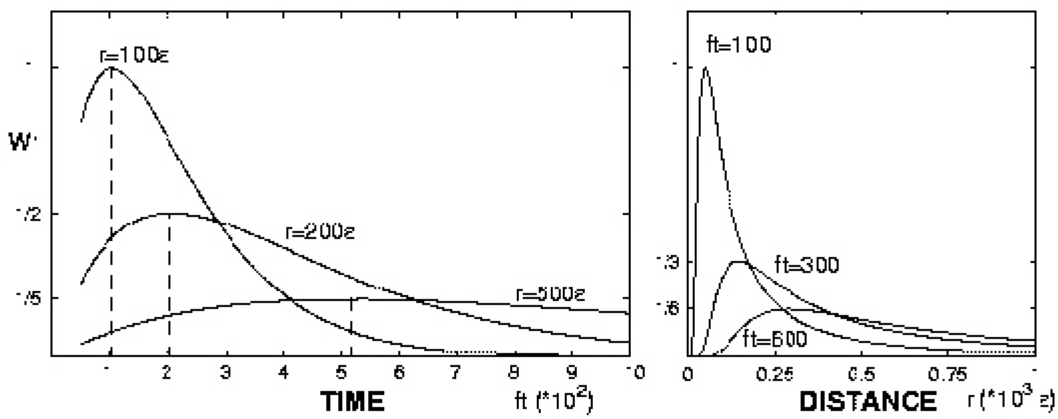
w_s-$\mathbf{G^n}$o u_n. Knowing the Green's functions, the response to arbitrary forcing can be determined numerically. In special cases the response can be expressed in closed form.

WORK COMPLETED

The vector Green's function $\mathbf{G^s}$ for the semi-infinite domain has been determined. It is such that numerical convolution with arbitrary surface forcing poses no problems. The Green's function $\mathbf{G^n}$ has to a large extent been determined. For the normal velocity component and the pressure things are almost complete. For the along-slope and upslope velocity components some more calculations are to be performed.

RESULTS

I have analyzed the response of a semi-infinite Ocean to `switch-on' point-sources and the response to finite-sized model surface. forcings. For finite-sized forcings I find that initially unbalanced geostrophic



Graphs showing the dispersing wave energy density (vertical axis) as a function of time (left) at three increasing distances from a finite-sized surface forcing on a constant frequency ray and (right) along a ray at three different times. This is for N=2f. ***Time is measured in inertial periods. ε is the horizontal size of the forcing, r is the distance from the forcing along the ray.***

currents are created which ultimately reach a steady state through radiation of internal waves. They propagate away from the forcing region as a distinct three-dimensional wavefield spreading throughout space. Amplitudes at a fixed point in space evolve as $t^{1/2}$exp(-a(x,y,z)t) with a(...)a positive function. In other words, the wavefield is a distinct pulse. At a fixed point in space it would appear that a wave packet passes by. The polarization relations for coherent linear wavefields do not hold. I find that for two very different initial conditions (unbalanced vortices) the same fraction of the initial total energy is converted into internal wave energy and that in both cases the energy spectrum of the internal wave field is the same. After the adjustment the geostrophic currents have increased in amplitude by a factor N/f. This means that if N is much larger than f, Rossby numbers increase dramatically (also the vertical shear increases). If the forcing is such that anti-cyclonic flow is generated, the final flow may be well inside the centrifugally unstable range (for more details see Kloosterziel, 2000). Centrifugal instability leads to further radiation of internal waves (see Carnevale et al., 1997). I am currently engaged in developing theory for centrifugal instability of baroclinic vortices in arbitrarily stratified rotating fluids. One remarkable result is that unlike in parallel shear flow in a non-rotating fluid, instability will depend not only on the sign of vorticity but also on the sign of the vertical shear (the Richardson criterion involves the magnitude of the shear). For the slope it has been determined that a brief normal flow leads to unbalanced currents which unlike at the surface will not reach an equilibrium but disappear through radiation of waves.

IMPACT/APPLICATIONS

Numerical codes for simulating the response to surface forcing or the reflection of wave packets off a slope can be tested for accuracy and errors by contrasting the output with the exact results I derive. A number of paradigms based on group velocity arguments and properties of monochromatic waves do not hold for wavefields with a continuum of wavevectors and frequencies. For example, the polarization relations for linear internal waves do not hold. Coherent wavefields generated by surface-forcings of the kind I used have horizontal current ellipses more elongated at each frequency than a single plane monochromatic wave would have. Deviations from the theoretical predictions using the polarization relations may possibly be used to determine how much of the internal wave field at a given location is truly random and how much is coherent. More generally this theoretical work will lead to a better understanding of the properties of non-random internal wave fields and currents near sloping topography.

RELATED PROJECTS

Dr. Carnevale of Scripps Institution of Oceanography, San Diego, is presently developing a code to study the reflection of internal wave packets at a slope. This code can also be used to simulate the internal wave generation due to surface pumping at an upper rigid surface. The quasi-analytical solutions I have and will research will be used by Carnevale to assess the performance of his code, while I will benefit from a comparison with his fully nonlinear simulations, that is, it will show to what extent the linear results are valid.

We also work together on the problem of centrifugal instability of barcoclinic vortices which can be a source of internal waves. My theory will be complemented by numerical simulations of unstable vortices by Carnevale.

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